Chapter 6
Currents in relation to auroral arcs

In chapter 4 we derived the following expression for the horizontal current in the high latitude ionosphere assuming that the magnetic field is pointing vertically downward:

$$\vec{j}_{\perp} = \sigma_P \vec{E}'_{\perp} + \sigma_H \frac{\vec{B} \times \vec{E}'_{\perp}}{B} \qquad (6.1)$$

where

$$E_{\perp} = \vec{E}_{\perp} + \vec{u} \times \vec{B}$$

and where $\vec{u}$ is the neutral wind.

We recall that the Pedersen current flow in the direction along the electric field while the Hall current is perpendicular to both the electric field and the magnetic field. In this chapter we will infer height integrated currents and consider the polarization effect in auroral arcs at night (non-sunlit).

6.1 Height integrated currents:

The horizontal currents perpendicular to the magnetic field is confined to a latitude range from say ~90 km to ~250 km. The bottom boundary of the conductive layer depends on the electron density profile which means solar ionization and particle impact ionization. However, it is rare that we have electron densities of significance below 90 km. The upper boundary is controlled by the mobility coefficients. Above 250 km electrons and ions both move in the same direction with the E x B – drift, i.e. they are confined to the magnetic field and hence not mobile across the magnetic field. Mobility across the magnetic field requires collisions with neutral constituents.

Integrating Equation 6.1 from 90 km to 250 km gives the following expression:

$$\vec{J}_{\perp} = \int_{90}^{250} \vec{j}_{\perp}\, dh = \int_{90}^{250} \sigma_P \vec{E}'_{\perp}\, dz + \int_{90}^{250} \sigma_H \frac{\vec{B} \times \vec{E}'_{\perp}}{B} dz \qquad (6.2)$$

The electric field penetrating down from the magnetosphere mapped and the Earth's magnetic field can be assumed unchanged over the actual altitude range. This allows us to write Eq. 6.2 on the form

$$\vec{J}_{\perp} = \Sigma_P \vec{E}'_{\perp} + \Sigma_H \frac{\vec{B} \times \vec{E}'_{\perp}}{B} \qquad (6.3)$$

where

$$\Sigma_P = \frac{e}{B} \int_{90}^{250} n_e k_P\, dz \quad \text{and} \quad \Sigma_H = \frac{e}{B} \int_{90}^{250} n_e k_H\, dz$$

are the height integrated conductivities for Pedersen and Hall currents, also referred to as Pedersen and Hall conductances. The electron density can be measured by radar and the mobility coefficients can be calculated. However, the critical point for such calculations is to have reliable models for the ion-neutral and the electron-neutral collision frequencies.

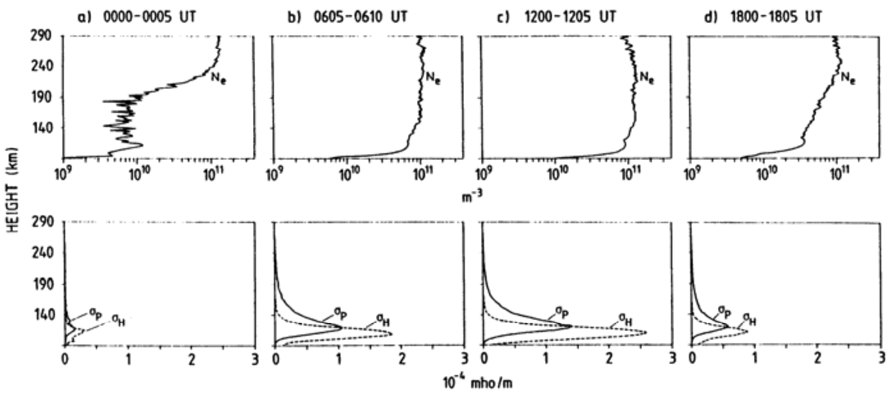


Figure 6.1 Upper panel: Four characteristic electron density profiles for midnight, morning midday and evening conditions on August 6-7, 1985 as observed by EISCAT UHF in Tromsø. Lower panel: Corresponding Hall- and Pedersen conductivity profiles (From Brekke and Hall, 1988).

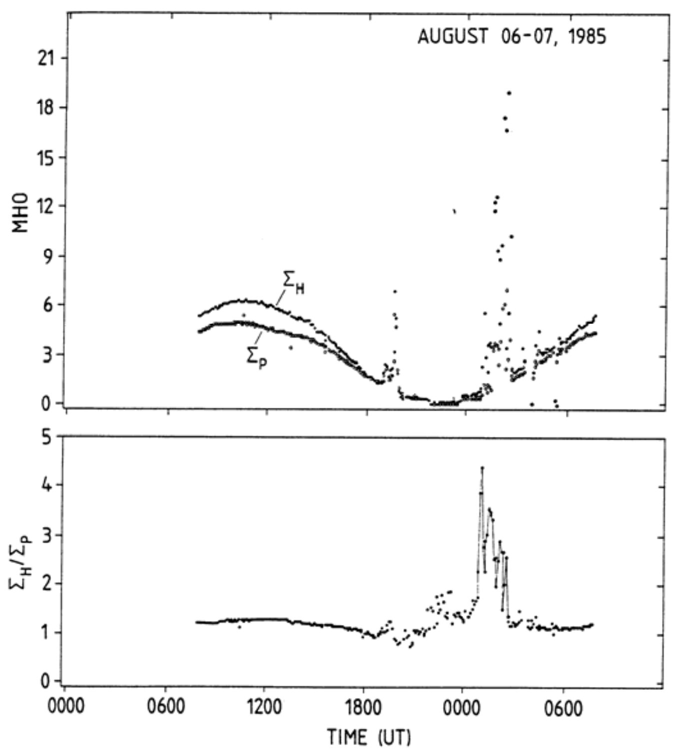


Figure 6.2 Upper panel: Hall- and Pedersen conductances as function of time on August 6-7, 1985 (campare with Figure 6.1). Lower panel: The corresponding Hall and Pedersen conductance ratio (From Brekke and Hall, 1988)

Introducing a local (x,y,z) coordinate system as shown in Figure 6.3 Equation 6.3 can be written in the form of a matrix:

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \begin{pmatrix} \Sigma_P & -\Sigma_H \\ \Sigma_H & \Sigma_P \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} \qquad (6.4)$$

where $\vec{B} = B\hat{z}$ and $\vec{E}_\perp = E_x\hat{x} + E_y\hat{y}$.

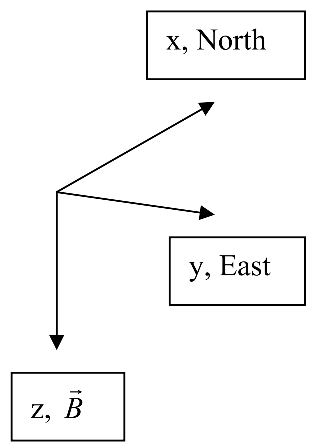


Figure 6.3 A local coordinate system where x is pointing geographic north, y east, and z is pointing vertically downward.

6.2 Polarization electric field within an auroral arc

We are now going to consider a simple model for the current configuration in association with an auroral arc at night. The auroral arc represents a stripe of strongly enhanced conductivity. Consider an ideal situation with an auroral arc of infinite length oriented in the east-west direction as shown in Figure 6.4.

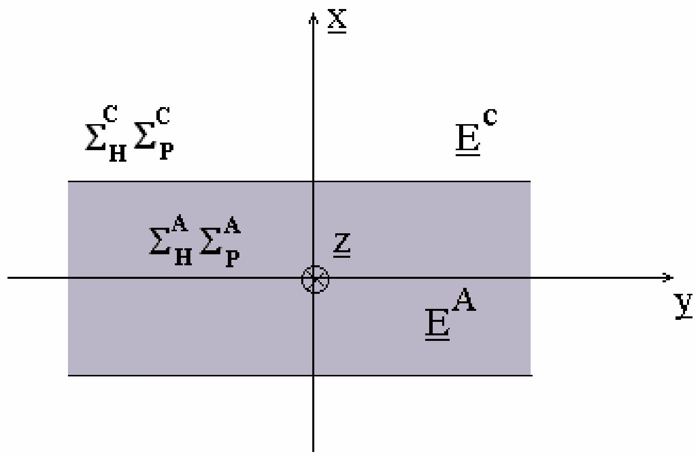


Figure 6.4 Auroral arc – an east-west elongated belt of strongly enhanced conductivity.

Assume a large scale electric field is mapped down from the magnetosphere onto the ionophere:

$$\vec{E} = \left(E_x, E_y, 0\right)$$

According to Faraday's law, when we assume that a stationary condition has achieved, the electric field is curl-free:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = 0 \text{ gives:}$$

$$x: \quad \frac{-\partial E_y}{\partial z} = 0$$

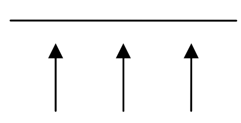
$$y: \quad \frac{\partial E_x}{\partial z} = 0$$

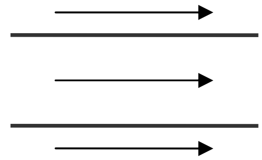
$$z: \quad \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0$$

Integrating the last equation (z-component) in the x-direction across the arc northern boundary we get:

$$\int_{E_y^A}^{E_y^C} \left(\frac{\partial E_y}{\partial x}\right) dx = \int \underbrace{\left(\frac{\partial E_x}{\partial y}\right)}_{=0} dx \qquad (6.5)$$

As the arc is infinitely long there is no change in the x-component along the y-direction





From Eq. 6.5 we then get that will not change across the arc boundary.

$$E_Y^C - E_Y^A = 0$$
$$E_Y^C = E_Y^A = E_y$$

Due to lack of solar EUV ionisation at night, the conductivity outside the arc is low and to a first assumption we neglect it. I.e.

$$\Sigma_P^C = \Sigma_H^C = 0$$

We further assume that the Birkeland current is zero:

$$j_{\|} = 0$$

Consider current continuity in x-direction. Outside the arc, $J_x^C = 0$, due to the total lack of conductivity. Since, $J_x^A + J_x^C = 0$, J_x^A has to be zero.

From the matrix in Equation (6.4) we have that:

$$J_x = \Sigma_P^A E_x^A + \Sigma_H^A E_y = 0$$
$$\Rightarrow E_x^A = -\frac{\Sigma_H^A}{\Sigma_P^A} E_y \qquad (6.6)$$

which is the polarization electric field
Inside the arc along y-direction:

$$J_y^A = -\Sigma_H^A E_x^A + \Sigma_P^A E_y^A \qquad (6.7)$$

Inserting Eq. 6.6 into 6.7 gives

$$J_y^A = \left[\frac{\left(\Sigma_H^A\right)^2}{\Sigma_P^A} + \Sigma_P^A \right] \cdot E_y \qquad (6.8)$$

In general $\Sigma_H^A >> \Sigma_P^A$ within a bright arc, i.e. the "auroral conductance" term in equation 6.8 can be strongly enhanced.

<u>Example:</u>

$$E_y = 100 mV/m$$
$$\Sigma_H^A = 30 mho$$
$$\Sigma_P^A = 10 mho$$

$$J = \left[\frac{900}{10} + 10\right] E_y = 100 E_y = 1A/m$$

I.e. an amplification factor of the conductance of a factor 10. Assuming that the arc is 1000 *km* wide, the total current across it is 10^6 A.

In this example we used a Hall to Pedersen conductance ratio of a factor 3. As demonstrated in Figure 6.5 it can be much higher.

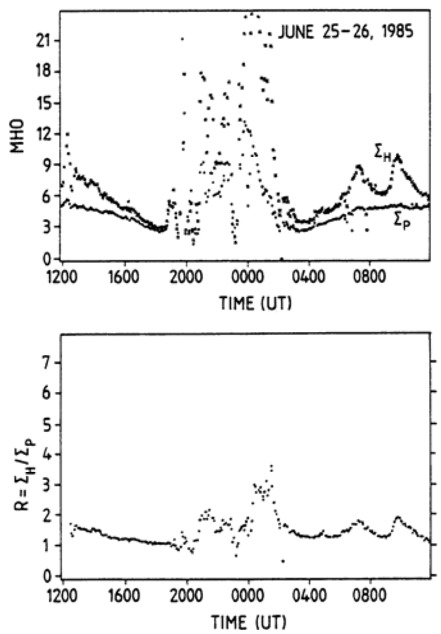
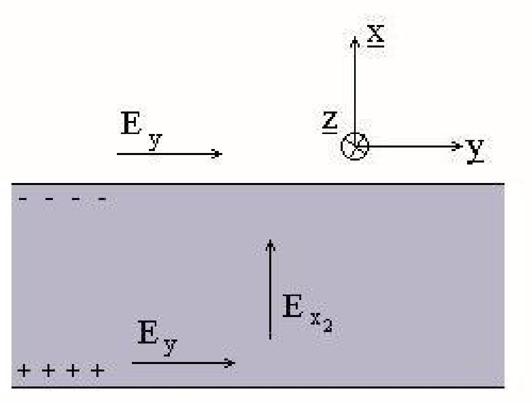


Figure 6.5: Ionsopheric coducatance data obtained by the EISCAT CP-1 experiment on June 25-26, 1985. Upper panel: The Hall and Pedersen conductances as function of universal time. Notice the two bumps in Σ_H in the morning sector on June 26, one before 08 UT and one after. These are likely due to energetic particle precipitation. Lower panel: The corresponding Hall to Pedersen conductance ratio. (From Moen et al., 1990).

6.2.1 A detailed consideration of the polarization effect



Primary current: assume $E_x = 0$
Hall current: $J_x = -\Sigma_H E_y$ gives rise to a secondary polarization field to build up

Pedersen current: $J_y = \Sigma_P E_y$

$J_{x_2} = \Sigma_P E_{x_2}$ → Secondary Pedersen current that cancels the primary J_H across the arc
$J_{y_2} = \Sigma_H E_{x_2}$ → secondary Hall current along the arc

6.3 A more realistic model with non-zero conductivities and $J_{\|}$

Let is now assume non-zero conductivity outside the arc and the presence of field-aligned currents. I.e.

$$\Sigma^C \neq 0$$
$$\bar{\mathrm{J}}_{\|} = 0$$

Current continuity in x-direction gives:

$$J_x^A = J_x^C + J_{\|} \qquad (6.9)$$

Eq. 6.4 for height integrated currents inside and outside of the arc in x-direction gives:

$$J_x^A = \Sigma_P^A E_x^A - \Sigma_H^A E_y^A \qquad (6.10)$$
$$J_x^C = \Sigma_P^C E_x^C - \Sigma_H^C E_y^C \qquad (6.11)$$

Inserting 6.10 and 6.11 in 6.9 gives:

$$\Sigma_P^A E_x^A - \Sigma_H^E E_y^A = \Sigma_P^E E_x^E - \Sigma_H^E E_y^E + J_{\|}$$

or expressed in terms of the polarization electric field:

$$E_x^A = \underbrace{\frac{\Sigma_P^C}{\Sigma_P^A} E_x^C + \frac{\left(\Sigma_H^A - \Sigma_H^C\right)}{\Sigma_P^A} E_y^C}_{E'} + \frac{J_{\|}}{\Sigma_P^A}$$

E' can be calculated if we know the applied $\vec{E}^C$ outside the arc, and the height integrated conductivities inside and outside the arc.

If $E_x^A \cong E'$ we have a polarization arc.

IF $E_x^A \cong \dfrac{J_{\|}}{\Sigma_P}$ we have a Birkeland current arc

6.4 Magnetosphere-Ionospheric current systems and magnetic deflections

6.4.1 Large scale convection

In the case of steady-state reconnection at day and night a steady state 2-cell convection pattern will establish as shown in Figure 6.6. Anti-sunward flow across the polar cap from day to night, and sunward return flow equatorward of that. In the F-region where frozen-in flux applies, the velocity vector is equivalent to an electric field vector:

$$\vec{v}_i = \frac{\overline{E} \times \vec{B}}{B^2} \qquad (6.12)$$

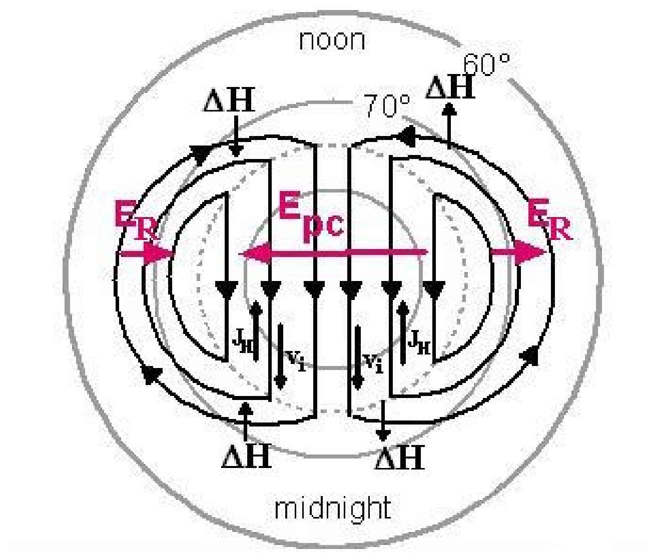


Figure 6.6 Standard two-cell polar cap convection system driven by solar wind-magetosphere interaction . See text for explanation.

The height integrated Hall current is given as:

$$\vec{J}_H = \Sigma_H \frac{\vec{B} \times \vec{E}}{B} \qquad (6.13)$$

Comparing 6.12 and 6.13 we see that

$$\vec{J}_H \sim -\vec{v}_i , \qquad (6.14)$$

i.e. an Hall current floats in the opposite direction of the ion velocity vectors, see Figure 6.6. Notice that cusp midday and midnight sectors can be identified as a polarity change in the H or X-component deflection observed by ground magnetometers. Also, due to the relationship given in Eq. 6.14 equivalent convection can be derived from ground magnetometers. This is most straight forward when the conductivity is homogenous but not straight forward in the case of intense auroral activity polarization arcs.

6.4.2 Magnetospheric currents

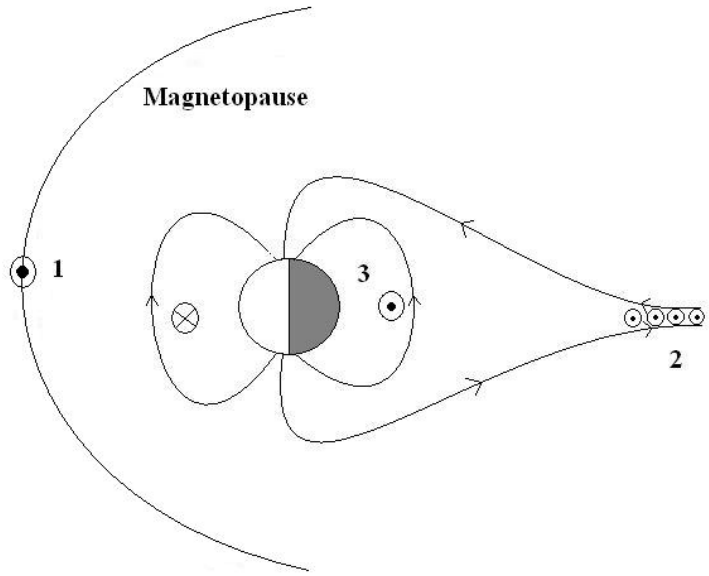


Figure 6.7

Figure 6.7 shows the location of three external current systems of the Earth's magnetosphere:

1: Magnetopause current (6-14 R_E). The eastward direction of this current can be argued from the Lorentz force ($\vec{F} = q\vec{v} \times \vec{B}$) separating solar wind electrons and ions at the magnetopause. Applying Amperes' law this current sheet gives rise to a strengthening of the Earth's magnetic field inside the magnetopause.

2: Tail current (20-1000 R_E). The direction of this current can be argued from the rotation in the Earth's magnetic field from tail ward below the current sheet to sunward above it (apply Amperes law: $\nabla \times \vec{B} = \mu_0 \vec{j}$)

3: Ring current (3-6R_E). Due to ∇B -drift and Curvature drift of electrons and ions injected from the tail during substorms.

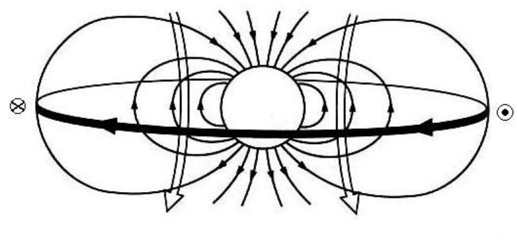
$$v_{\nabla B} = \frac{1}{2} m v_\perp^2 \frac{\vec{B} \times \nabla \vec{B}}{qB^3}$$

$$v_{cf} = \frac{m v_\parallel^2}{qB^2} \frac{\vec{R}_L \times \vec{B}}{R_L^2}$$

6.4.3 Ground magnetic deflections

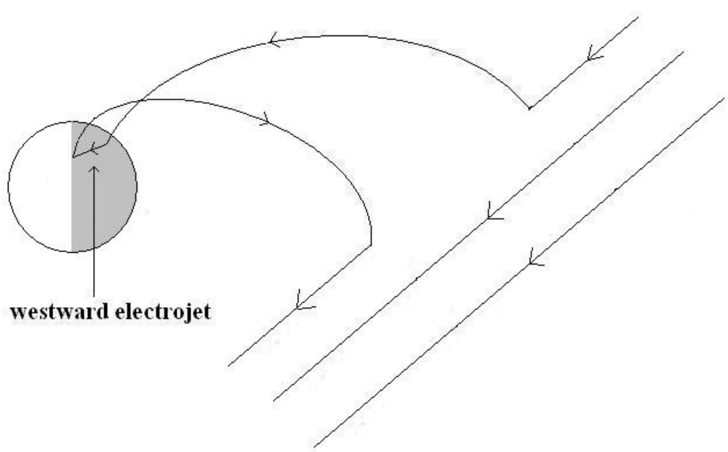
1: Magnetopause-current gives rise to $+\Delta X$ (or ΔH) near the equator

2: Ring current



$-\Delta X$ near the equator

3. Tail current



ΔX negative at auroral latitudes.